REVIEWS

Studies in Non-Linear Stability Theory. By WIKTOR ECKHAUS. Springer-Verlag, 1965. 117 pp. DM 22.

This monograph discusses the stability of steady or periodic solutions of certain kinds of non-linear partial differential equations. Although fluid-mechanical applications are omnipresent, they intrude but little into this mathematical account. The book, therefore, may be of interest mainly to the student of non-linear equations, perturbation methods, eigenfunction expansions and so on, whether or not he has an interest in fluid mechanics. It is also relevant to say that the book is largely concerned with, and is firmly based on, Dr Eckhaus's own researches, which were published in French during 1962–63 in *Journal de Mécanique*. It is of value to have his work put on record for English-speaking readers.

The differential equations considered are of the type which occur in non-linear hydrodynamic stability, but contain derivatives in one or two spatial dimensions only (not three). Moreover, time appears only as a first derivative. These two restrictions have the consequence that the treatment perforce ignores two or three important areas, where non-linear stability theory has been applied with some success, namely, in cases of thermal or centrifugal instability (for which the equations contain second differentials with respect to time, and may involve three space dimensions), and in cases of three-dimensional perturbations to two-dimensional parallel flows. On the other hand such problems have been treated by others by other perturbation schemes, effective within their ranges of application. Several of the relevant papers are mentioned briefly, but one substantial paper not at all, namely, A. Davey's quantitative comparison between theory and experiment for Taylor-vortex flows between rotating cylinders. In the absence of a rigorous mathematical theory such comparisons are desirable and would have given the reader greater confidence in the relevance of non-linear theories of this type.

The problem with which Dr Eckhaus is concerned can be described as follows: the non-linear differential equation is known to have an equilibrium solution or state, whose stability can be examined by solution of the linearized equation for small perturbations from that state. Stability or instability is thus known within delineated ranges of the parameters occurring in the equations; the question is then asked of the influence of the non-linearity on the growth or decay of perturbation in time. In particular, do other, stable or unstable, equilibrium states arise? The author studies this problem, first, for a class of non-linear partial differential equations with one space variable, whose linearized-stability (eigenvalue) problems may or may not be of Sturm–Liouville type. If a given case is, an eigenfunction expansion theorem is known; if it is not, such a theorem is assumed to be true. In either case the author expands the non-linear solution in a series of linearized-theory eigenfunctions with time-dependent amplitudes. The analysis yields a set of ordinary differential equations for the amplitudes; these are discussed at length, but with little reference to standard work on the subject. Applications to some model equations, including that of J. M. Burgers, are described.

In the final chapters of the book the author considers a class of non-linear partial differential equations in two spatial variables; and after Fourier analysis in one of the latter, an eigenfunction expansion again is assumed. As before the analysis leads to a set of ordinary non-linear differential equations for the time-dependent amplitudes. For values of a parameter ('Reynolds number') just above the critical for small perturbations, solutions periodic (and nearly sinusoidal) in one spatial variable (and perhaps in time also) are possible. The amplitude attained depends on the wave-number, and it is important to know the 'relative' stability of finite-amplitude disturbances of different wave-numbers. The author shows that, at a given 'Reynolds number', a finite-amplitude solution is stable if its wave-number k lies in a certain range about the wave-number of maximum amplification, in the sense that small periodic perturbations with different k decay to zero (even though they would be amplified according to linearized theory). The range of k does not extend to the two branches of the neutral curve. This result, which is derived for a general class of equations, is related to some work of L. A. Segel (1962) on the thermal convection problem.

After applying his method to a model equation, the author describes an application to plane Poiseuille flow, subject to two-dimensional perturbations, and derives a form of solution which is the same as that given elsewhere by other methods. Detailed numerical calculations are not presented.

As a formal scheme Dr Eckhaus's work has much to recommend it, in spite of difficulties in application due to a lack of knowledge of a complete set of eigenfunctions of linearized-stability theory. It is to be hoped that his ideas will be applied to other problems.

J. T. STUART

Research Inspired by the Dutch Windmills. By the 'PRINSENMOLEN' COMMITTEE. H. Veenman en Zonen. N.D. 184 pp. 30*s*. or $4.00.

The Prinsenmolen windmill is situated north of Rotterdam. It was bought by the Schieland Catchment Board in order to preserve it, not as a useless relic, but as a means of draining the neighbouring polders. An influential committee was set up in 1936 to modernize and test the mill, and their work expanded over a long period of years to include a much wider field.

This book, which is written throughout in excellent English, gives an account of their labours. It is composed with a wealth of detail like the *Philosophical Transactions* of (say) sixty years ago, so that the reader is able to comprehend fully the problems involved and to judge for himself the success of the means that could be taken to solve them. The style is thus in pleasing contrast to the average paper of today, which suppresses much of the practical detail on which the validity of the conclusions depends.

Chapters I and II are historical. The account starts in early Renaissance times because before 1500 an important part of the Low Countries had sunk below sea level. Even in the nineteen-twenties mill gearing remained very simple. Commonly the main wooden shaft, on which the sails were mounted, was slightly inclined to the horizontal and carried a brake wheel provided with wooden cogs. These worked against the staves of a lantern pinion (called a wallower) at the top of the vertical shaft. Tests in 1926 on the Nienwland mill were carried out, which were notable for the use of an ingenious dynamometer. This determined the load on the cogs from a measurement of the lateral force exerted on its housing by the top bearing of the vertical shaft. An enormous effort was made in wind tunnels and on mills themselves to improve the sails, and the water wheels too received much attention.

Chapters III–V begin the description of the Committee's work. The Nienwland experiments were repeated on the Prinsenmolen mill, of which full details are given. The span of its sails was 28·7 m. A typical test showed that 35 h.p. at 17 r.p.m. was developed in the main shaft, the force exerted by the cogs being 1080 kg. But only 12 h.p. was usefully exerted in raising water. It turned out that the principal difficulty in obtaining good readings was the unsteadiness of the wind, although the mill was exceptionally well sited with a large expanse of water to windward. A considerable reduction in the losses was achieved by modifying the scoop wheel and the water channels, but attempts to use a Prony brake on the main shaft were a failure. At this stage suspicion had fallen on the cog measurements, and accurate comparative tests of different kinds of sails could not be made.

Chapters VI–VIII explain how this difficulty was overcome by constructing a small mill of span 7·20 m. The steel main shaft ran on ball bearings, and it was possible to fit a satisfactory Prony brake. The first sails tried were of an old Dutch design. Tests proved difficult owing to irregular rotation, and much better results were obtained after 15 kg of lead had been fixed to the tip of each of the four sails, thus doubling the moment of inertia of the rotating parts. After aesthetic objections and workmen's prejudices had been overcome, sails shaped like airscrew blades were tried and proved to be far superior to the Dutch sails. They were of Russian design, and appeared to be based on a similar theory to that devised in England by the Aeronautical Research Committee. The measured and theoretical aerodynamic torques were found to be nearly the same, thus sail design should always make use of airscrew theory.

Wind tunnel tests on models of Dutch sails are described in Chapters IX and X. Further full-scale work was done at Prinsenmolen in 1939 with a new set of sails (XI) and on other large drainage mills (XII). The structure of the wind at different heights and places in Holland was elaborately examined (XIII), and the book ends with a discussion of interference caused to a mill by another lying to windward. This chances to be of topical interest to English engineers in view of the disaster at Ferrybridge, where a group of cooling towers were severely damaged in a gale. The Dutch rule is that interference is negligible as long as the distance apart of the mills exceeds ten times the sail span. The Committee's work showed that this distance is inadequate. There is also a brief account of electricity generation by windmills.

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